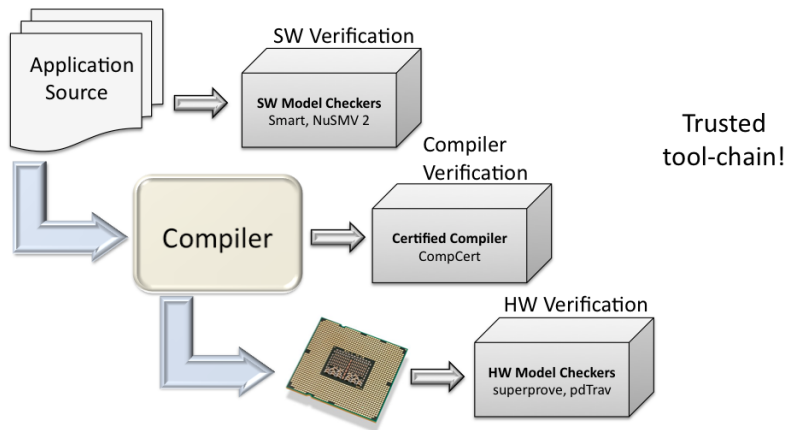


# Who Watches the Watchers: Toward Provably-correct Decision Diagram Code

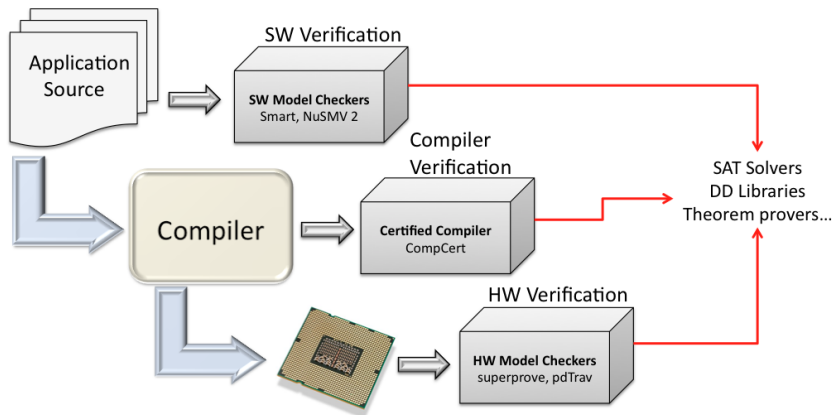
**Yousra Lembachar**, Ryan Rusich,  
Iulian Neamtiu, Gianfranco Ciardo

University of California, Riverside

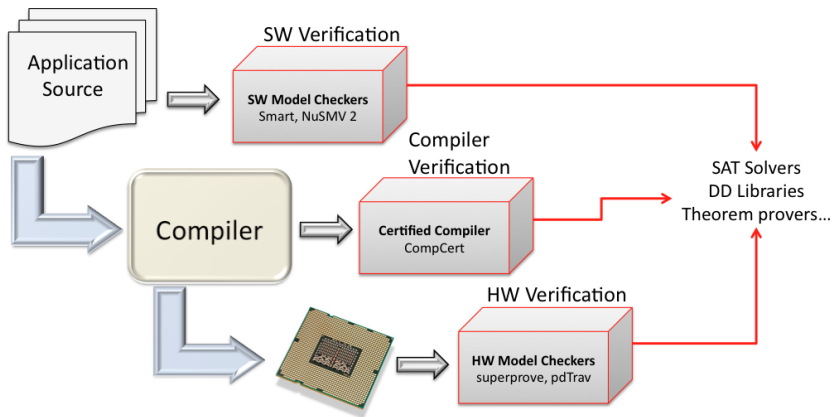
# Toward a Completely Verified Software Toolchain



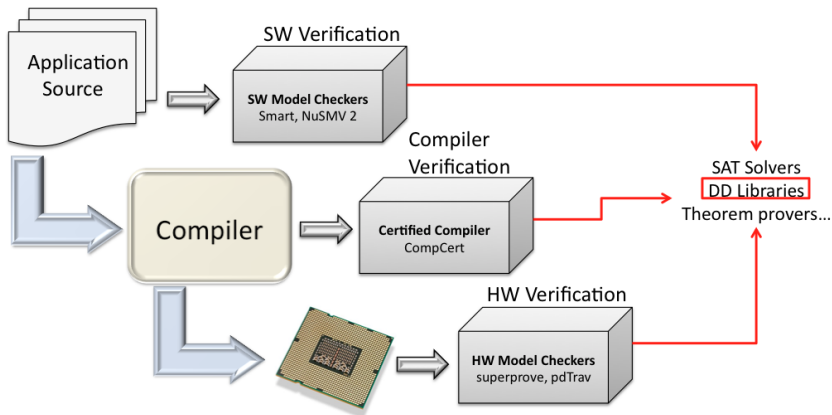
# Toward a Completely Verified Software Toolchain



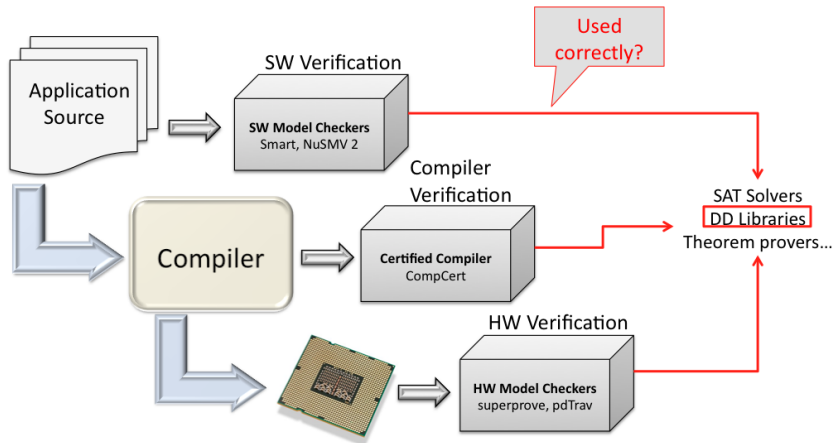
# Toward a Completely Verified Software Toolchain



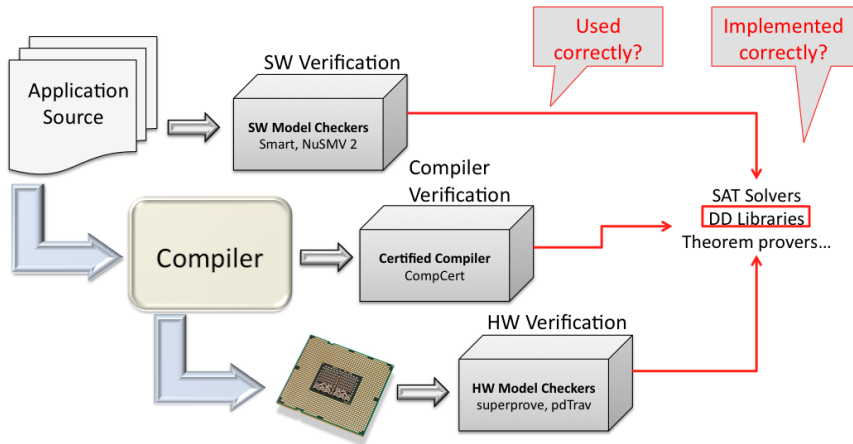
# Toward a Completely Verified Software Toolchain



# Toward a Completely Verified Software Toolchain



# Toward a Completely Verified Software Toolchain



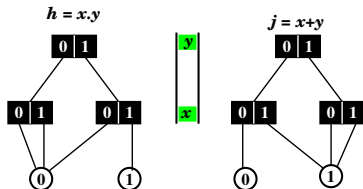
# Our Contribution

- ▶ BDDL
  - ▶ A calculus for reasoning about decision diagram library **and** client code
  - ▶ Provides a sound type system with operational semantics
  - ▶ Enables compile-time detection of dynamic errors
  - ▶ Enforces correct structural properties and semantics for decision diagrams
- ▶ Demonstrate the efficacy of our approach via real world bugs detected in three mature libraries: CUDD (NuSMV), MDDL (SMART), and JavaBDD



# Runtime Error Example in CUDD

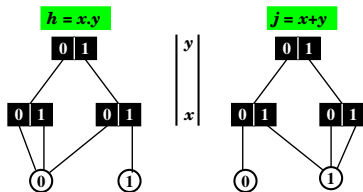
```
int main () {  
    Cudd mgr(0,2);  
    BDD x = mgr.bddVar();  
    BDD y = mgr.bddVar();  
    BDD h = x * y;  
    BDD j = x + y;  
    BDD k = h.Compose(j,2);  
}
```



```
DdNode* Cudd_bddCompose(  
    DdManager * dd, DdNode * f,  
    DdNode * g, int v) {  
    DdNode *proj, *res;  
    /* Sanity check. */  
    if (v < 0 || v >= dd->size)  
        return(NULL);  
    proj =dd->vars[v];  
    do {  
        ...  
    } while (dd->reordered == 1);  
    return(res);}  
BDD BDD::Compose(BDD g, int v)  
{ ...  
    return BDD(...,  
        Cudd_bddCompose(  
            mgr.node, g.node, v));  
}
```

# Runtime Error Example in CUDD

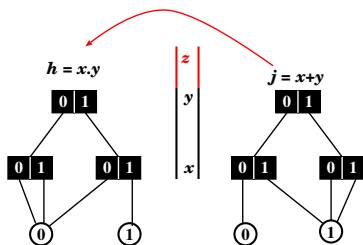
```
int main () {  
    Cudd mgr(0,2);  
    BDD x = mgr.bddVar();  
    BDD y = mgr.bddVar();  
    BDD h = x * y;  
    BDD j = x + y;  
    BDD k = h.Compose(j,2);  
}
```



```
DdNode* Cudd_bddCompose(  
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    DdNode *proj, *res;  
    /* Sanity check. */  
    if (v < 0 || v >= dd->size)  
        return(NULL);  
    proj =dd->vars[v];  
    do {  
        ...  
    } while (dd->reordered == 1);  
    return(res);}  
BDD BDD::Compose(BDD g, int v)  
{ ...  
    return BDD(...,  
        Cudd_bddCompose(  
            mgr.node, g.node, v));  
}
```

# Runtime Error Example in CUDD

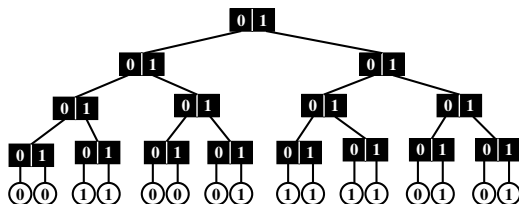
```
int main () {  
    Cudd mgr(0,2);  
    BDD x = mgr.bddVar();  
    BDD y = mgr.bddVar();  
    BDD h = x * y;  
    BDD j = x + y;  
    BDD k = h.Compose(j,2); X  
}
```



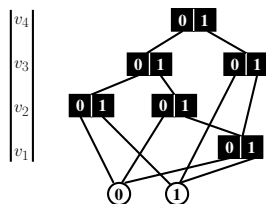
```
DdNode* Cudd_bddCompose(  
    DdManager * dd, DdNode * f,  
    DdNode * g, int v) {  
    DdNode *proj, *res;  
    /* Sanity check. */  
    if (v < 0 || v >= dd->size)  
        return(NULL);  
    proj = dd->vars[v];  
    do {  
        ...  
    } while (dd->reordered == 1);  
    return(res);}  
BDD BDD::Compose(BDD g, int v)  
{ ...  
    return BDD(...,  
        Cudd_bddCompose(  
            mgr.node, g.node, v));  
}
```

# Binary Decision Diagrams (BDDs)

Binary tree



BDD

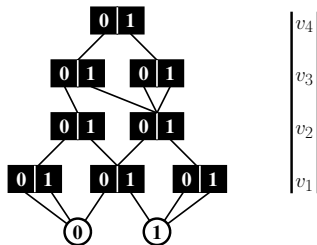


$$f(v_1, v_2, v_3, v_4) = (v_4 \vee v_2) \wedge (v_3 \rightarrow v_1)$$

# BDD Encodings - Sets and n-ary relations

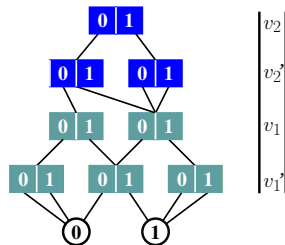
Set

{0011, 0101, 0110, 0111, 1001,  
1010, 1011, 1101, 1110, 1111}



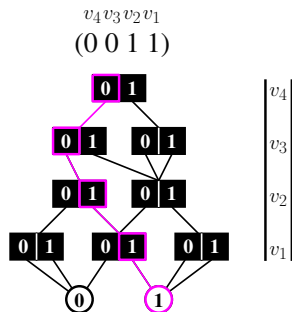
Binary relation

{(01,01) (00,11) (01,10) (01,11)  
(10,01) (11,00) (11,01) (10,11)  
(11,10) (11,11)}

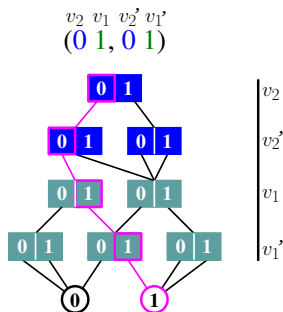


# BDD Encodings - Sets and n-ary relations

Set

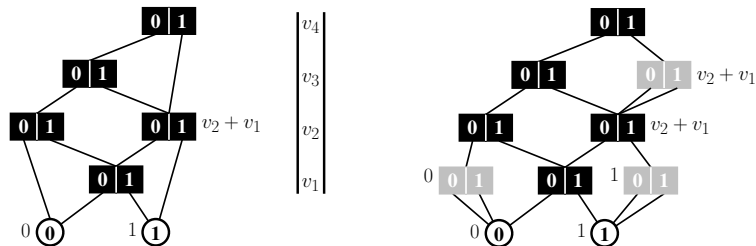


Binary relation



# BDD Reduction Rules

Reduction rules for canonicity and compactness...



Fully-reduced (left) vs. quasi-reduced (right) BDDs

# BDDL Calculus

## Terms

```

t ::=
  v |  $\times$ 
  | succ t | pred t
  | iszero t
  | t t
  | letrec x :  $\tau$  = t in t
  | if then t else t t
  | ref t | !t
  | Bnode (t, i, t, t, t)
  | t.level | t.index
  | t.var | t.tchild
  | t.fchild

```

## Types

```

 $\tau$  ::=
  bool | nat
  | string | Id
  |  $\tau \rightarrow \tau$ 
  | ref  $\tau$ 
  | bdd[l, r, c]
  |  $\{\nu : \tau \mid \rho(\pi)\}$ 

```

## Typing rules (example)

$$\frac{\begin{array}{l} \Gamma \vdash id : Id \quad id \notin dom(\Gamma) \quad \Gamma \vdash v_{var} : string \\ \Gamma \vdash t_0 : \{\nu : nat \mid \nu \geq 1 \wedge \nu = l\} \\ \Gamma \vdash t_1 : ref \ bdd[l', r, c] \\ bdd[l', r, c] <:_B \{\nu : bdd[p, r, c] \mid l = p + 1\} \\ \Gamma \vdash t_2 : ref \ bdd[l'', r, c] \\ bdd[l'', r, c] <:_B \{\nu : bdd[p', r, c] \mid l = p' + 1\} \end{array}}{\Gamma, id : Id \vdash Bnode (t_0, id, v_{var}, t_1, t_2) : bdd[l, r, c]} \text{ (T-BNODE)}$$

## Operational semantics (example)

$$\frac{\mu; \mathbf{t}_j \longrightarrow \mu'; \mathbf{t}'_j}{\mu; Bnode(v_i^{i \in 1, \dots, j-1}; \mathbf{t}_j; \mathbf{t}_k^{k \in j+1, \dots, 5}) \longrightarrow \mu'; Bnode(v_i^{i \in 1, \dots, j-1}; \mathbf{t}'_j; \mathbf{t}_k^{k \in j+1, \dots, 5})} \text{ (E-BNODE)}$$

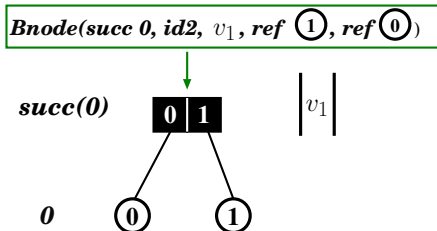
## Soundness proof

A well-typed BDD program can't go wrong.



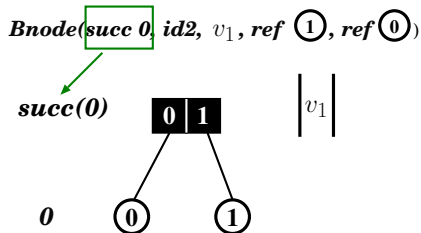
# BDDL Terms - The Bnode term and its attributes

```
t ::=
  v | x
  | succ t | pred t
  | iszero t
  | λ:τ.t
  | t t
  | letrec x:τ = t in t
  | if then t else t t
  | ref t | !t
  | Bnode (t, i, t, t, t)
  | t.level | t.index
  | t.var | t.tchild
  | t.fchild
```



# BDDL Terms - The Bnode term and its attributes

```
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  | t.level | t.index
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  | t.fchild
```

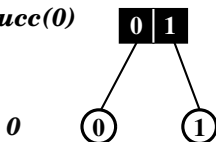


# BDDL Terms - The Bnode term and its attributes

```
t ::=  
  v | x  
  | succ t | pred t  
  | iszero t  
  | λ:τ.t  
  | t t  
  | letrec x:τ = t in t  
  | if then t else t t  
  | ref t | !t  
  | Bnode (t, i, t, t, t)  
  | t.level | t.index  
  | t.var | t.tchild  
  | t.fchild
```

$Bnode(succ\ 0, id2, v_1, ref\ ①, ref\ ②)$

$succ(0)$

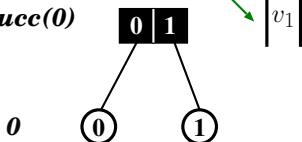


# BDDL Terms - The Bnode term and its attributes

```
t ::=
  v | x
  | succ t | pred t
  | iszero t
  | λ:τ.t
  | t t
  | letrec x:τ = t in t
  | if then t else t t
  | ref t | !t
  | Bnode (t, i, t, t, t)
  | t.level | t.index
  | t.var | t.tchild
  | t.fchild
```

$Bnode(succ\ 0, id2, v_1, ref\ ①, ref\ ②)$

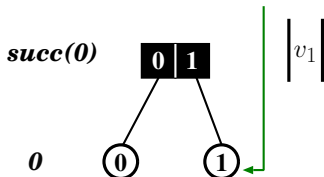
$succ(0)$



# BDDL Terms - The Bnode term and its attributes

```
t ::=  
  v | x  
  | succ t | pred t  
  | iszero t  
  | λ:τ.t  
  | t t  
  | letrec x:τ = t in t  
  | if then t else t t  
  | ref t | !t  
  | Bnode (t, i, t, t, t)  
  | t.level | t.index  
  | t.var | t.tchild  
  | t.fchild
```

$Bnode(succ\ 0, id2, v_1, \text{ref } \textcircled{1}, \text{ref } \textcircled{0})$

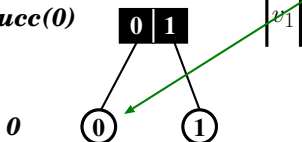


# BDDL Terms - The Bnode term and its attributes

```
t ::=  
  v | x  
  | succ t | pred t  
  | iszero t  
  | λ:τ.t  
  | t t  
  | letrec x:τ = t in t  
  | if then t else t t  
  | ref t | !t  
  | Bnode (t, i, t, t, t)  
  | t.level | t.index  
  | t.var | t.tchild  
  | t.fchild
```

***Bnode(succ 0, id2, v<sub>1</sub>, ref ①, ref ②)***

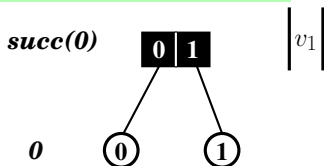
***succ(0)***



# BDDL Terms - $\lambda$ -calculus based terms

```
t ::=  
v | x  
| succ t | pred t  
| iszero t  
|  $\lambda:\tau.t$   
| t t  
| letrec x: $\tau$  = t in t  
| if then t else t t  
| ref t | !t  
| Bnode (t, i, t, t, t)  
| t.level | t.index  
| t.var | t.tchild  
| t.fchild
```

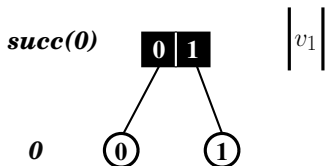
```
letrec v1 = ... in  
letrec id2 = ... in  
letrec build =  
   $\lambda n .$  /* n = succ(0)*/  
   $\lambda b .$  /* b = true*/  
    if iszero n then  
      (if b then  $\boxed{1}$  else  $\boxed{0}$ )  
    else  
      Bnode (succ(0), id2, v1,  
            build 0 true, build 0 false)  
in build (succ(0) true)
```



# BDDL Terms - $\lambda$ -calculus based terms

```
t ::=
  v | x
| succ t | pred t
| iszero t
|  $\lambda:\tau.t$ 
| t t
| letrec  $x:\tau = t$  in t
| if then t else t t
| ref t | !t
| Bnode (t, i, t, t, t)
| t.level | t.index
| t.var | t.tchild
| t.fchild
```

```
letrec  $v_1 = \dots$  in
letrec  $id_2 = \dots$  in
letrecbuild =
   $\lambda n.$     /*  $n = \text{succ}(0)$  */
   $\lambda b.$     /*  $b = \text{true}$  */
    if iszero n then
      (if b then  $\boxed{1}$  else  $\boxed{0}$ )
    else
      Bnode (succ(0),  $id_2$ ,  $v_1$ ,
             build 0 true, build 0 false)
  in build (succ(0) true)
```

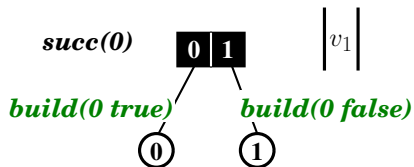




# BDDL Terms - $\lambda$ -calculus based terms

```
t ::=
  v | x
| succ t | pred t
| iszero t
|  $\lambda:\tau.t$ 
| t t
| letrec  $x:\tau = t$  in t
| if then t else t t
| ref t | !t
| Bnode (t, i, t, t, t)
| t.level | t.index
| t.var | t.tchild
| t.fchild
```

```
letrec  $v_1 = \dots$  in
letrec  $id_2 = \dots$  in
letrecbuild =
   $\lambda n.$ 
   $\lambda b.$ 
  if iszero n then
    (if b then  $\boxed{1}$  else  $\boxed{0}$ )
  else
    Bnode (succ(0),  $id_2$ ,  $v_1$ ,
           build 0 true, build 0 false)
  in build (succ(0) true)
```



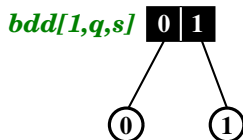
# BDDL Types

```
 $\tau :=$   
  bool | nat  
  | string | Id  
  |  $\tau \rightarrow \tau$   
  | ref  $\tau$   
  | bdd[l,r,c]  
  |  $\nu : \tau$  |  $\rho(\pi)$ 
```

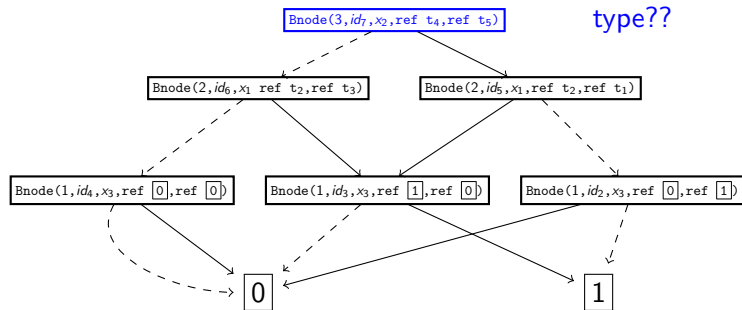
```
l :=  $\perp$ , nv  
r :=  $\perp$ , f, q  
c :=  $\perp$ , s, e
```

```
 $\pi := \nu$  | l | r | c
```

```
letrec  $v_1$  : string = ... in  
letrec  $id_2$  : Id = ... in  
letrec build :  
  { $\nu : nat$  |  $\nu = l$ }  $\rightarrow$  bool  $\rightarrow$  bdd[l, r, c] |  $l \leq 1$   
 $\lambda$  n .  
   $\lambda$  b .  
    if iszero n then  
      (if b then  $\boxed{1}$  else  $\boxed{0}$ )  
    else  
      Bnode (succ(0),  $id_2$ ,  $v_1$ ,  
        build 0 true, build 0 false)  
  in build (succ(0) true)
```

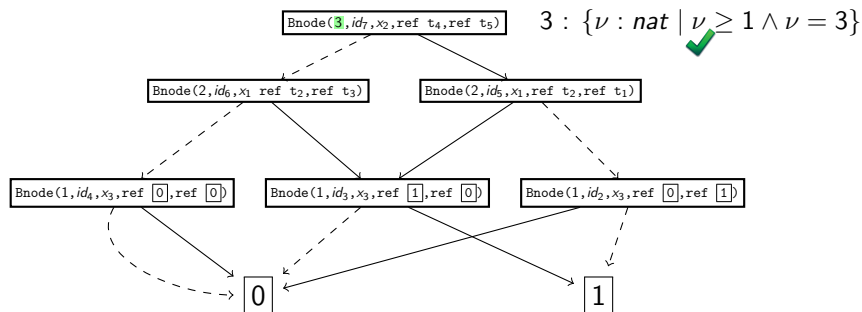


# Type checking and type inference for a 3-level BDD



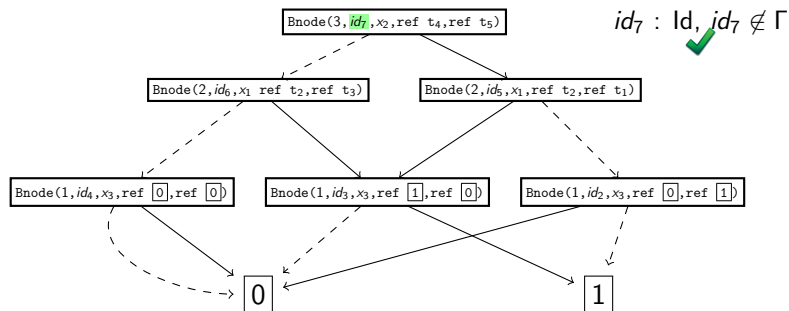
$$\text{T-BNODE} \frac{
 \begin{array}{l}
 \Gamma \vdash id : Id \quad id \notin \text{dom}(\Gamma) \quad \Gamma \vdash v_{var} : \text{string} \quad \Gamma \vdash t_0 : \{\nu : \text{nat} \mid \nu \geq 1 \wedge \nu = l\} \\
 \Gamma \vdash t_1 : \text{ref } bdd[l', r, c] \quad bdd[l', r, c] <:_B \{\nu : bdd[p, r, c] \mid l = p + 1\} \\
 \Gamma \vdash t_2 : \text{ref } bdd[l'', r, c] \quad bdd[l'', r, c] <:_B \{\nu : bdd[p', r, c] \mid l = p' + 1\}
 \end{array}
 }{
 \Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : bdd[l, r, c]
 }$$

# Type checking and type inference for a 3-level BDD



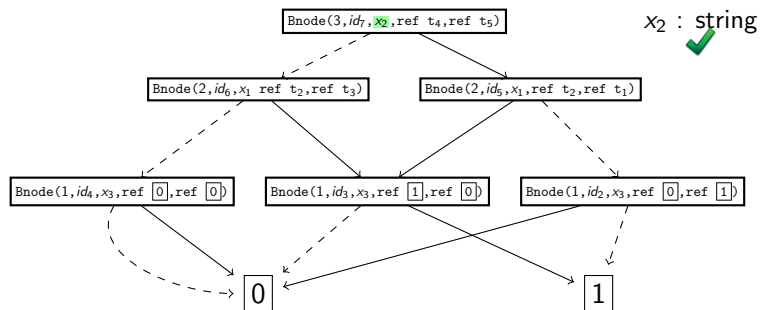
$$\begin{array}{l}
 \Gamma \vdash id : Id \quad id \notin \text{dom}(\Gamma) \quad \Gamma \vdash v_{var} : string \quad \Gamma \vdash t_0 : \{\nu : nat \mid \nu \geq 1 \wedge \nu = l\} \\
 \Gamma \vdash t_1 : \text{ref } bdd[l', r, c] \quad bdd[l', r, c] <:_B \{\nu : bdd[p, r, c] \mid l = p + 1\} \\
 \Gamma \vdash t_2 : \text{ref } bdd[l'', r, c] \quad bdd[l'', r, c] <:_B \{\nu : bdd[p', r, c] \mid l = p' + 1\} \\
 \text{T-BNODE} \frac{}{\Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : bdd[l, r, c]}
 \end{array}$$

# Type checking and type inference for a 3-level BDD



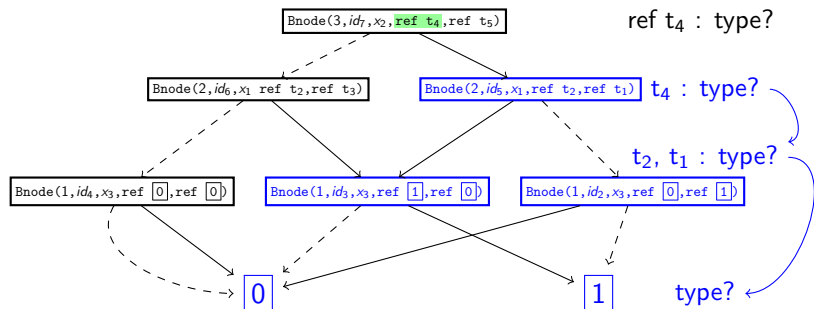
$$\text{T-BNODE} \frac{
 \begin{array}{l}
 \Gamma \vdash id : Id \quad id \notin \text{dom}(\Gamma) \quad \Gamma \vdash v_{var} : string \quad \Gamma \vdash t_0 : \{\nu : nat \mid \nu \geq 1 \wedge \nu = l\} \\
 \Gamma \vdash t_1 : \text{ref } bdd[l', r, c] \quad bdd[l', r, c] <:_B \{\nu : bdd[p, r, c] \mid l = p + 1\} \\
 \Gamma \vdash t_2 : \text{ref } bdd[l'', r, c] \quad bdd[l'', r, c] <:_B \{\nu : bdd[p', r, c] \mid l = p' + 1\}
 \end{array}
 }{
 \Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : bdd[l, r, c]
 }$$

# Type checking and type inference for a 3-level BDD



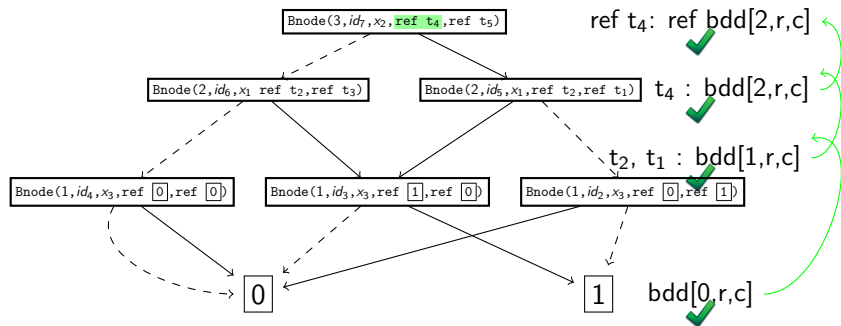
$$\begin{array}{c}
 \Gamma \vdash id : Id \quad id \notin \text{dom}(\Gamma) \quad \Gamma \vdash v_{var} : \text{string} \quad \Gamma \vdash t_0 : \{\nu : \text{nat} \mid \nu \geq 1 \wedge \nu = l\} \\
 \Gamma \vdash t_1 : \text{ref } bdd[l', r, c] \quad bdd[l', r, c] <:_B \{\nu : bdd[p, r, c] \mid l = p + 1\} \\
 \Gamma \vdash t_2 : \text{ref } bdd[l'', r, c] \quad bdd[l'', r, c] <:_B \{\nu : bdd[p', r, c] \mid l = p' + 1\} \\
 \text{T-BNODE} \frac{}{\Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : bdd[l, r, c]}
 \end{array}$$

# Type checking and type inference for a 3-level BDD



$$\begin{array}{l}
 \Gamma \vdash id : Id \quad id \notin dom(\Gamma) \quad \Gamma \vdash v_{var} : string \quad \Gamma \vdash t_0 : \{\nu : nat \mid \nu \geq 1 \wedge \nu = l\} \\
 \Gamma \vdash t_1 : ref \quad bdd[l', r, c] \quad bdd[l', r, c] <:_B \{\nu : bdd[p, r, c] \mid l = p + 1\} \\
 \Gamma \vdash t_2 : ref \quad bdd[l'', r, c] \quad bdd[l'', r, c] <:_B \{\nu : bdd[p', r, c] \mid l = p' + 1\} \\
 \text{T-BNODE} \frac{}{\Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : bdd[l, r, c]}
 \end{array}$$

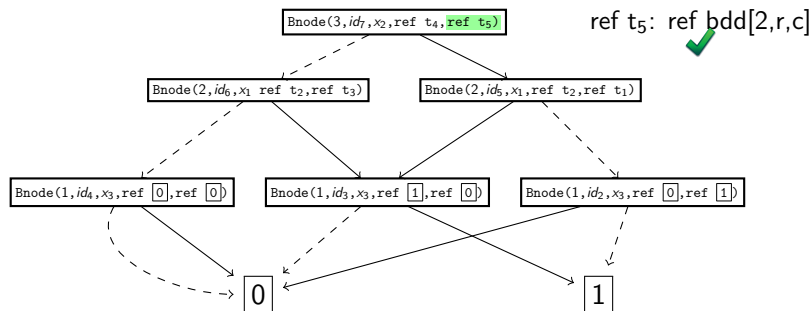
# Type checking and type inference for a 3-level BDD



$$\begin{array}{l}
 \Gamma \vdash id : Id \quad id \notin dom(\Gamma) \quad \Gamma \vdash v_{var} : string \quad \Gamma \vdash t_0 : \{\nu : nat \mid \nu \geq 1 \wedge \nu = l\} \\
 \Gamma \vdash t_1 : ref \text{ bdd}[l', r, c] \quad bdd[l', r, c] <:_B \{\nu : bdd[p, r, c] \mid l = p + 1\} \\
 \Gamma \vdash t_2 : ref \text{ bdd}[l'', r, c] \quad bdd[l'', r, c] <:_B \{\nu : bdd[p', r, c] \mid l = p' + 1\} \\
 \text{T-BNODE} \frac{}{\Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : bdd[l, r, c]}
 \end{array}$$

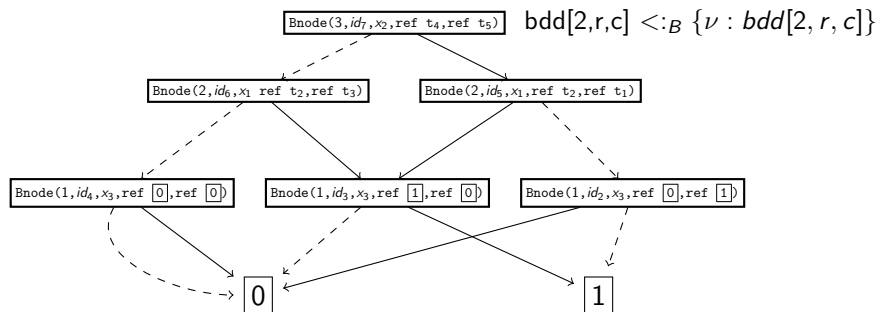


# Type checking and type inference for a 3-level BDD



$$\begin{array}{c}
 \Gamma \vdash id : Id \quad id \notin \text{dom}(\Gamma) \quad \Gamma \vdash v_{var} : \text{string} \\
 \Gamma \vdash t_1 : \text{ref } bdd[l', r, c] \quad bdd[l', r, c] <_B \{ \nu : bdd[p, r, c] \mid l = p + 1 \} \\
 \Gamma \vdash t_2 : \text{ref } bdd[l'', r, c] \quad bdd[l'', r, c] <_B \{ \nu : bdd[p', r, c] \mid l = p' + 1 \} \\
 \hline
 \text{T-BNODE} \quad \Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : bdd[l, r, c]
 \end{array}$$

# Type checking and type inference for a 3-level BDD



$\Gamma \vdash id : Id \quad id \notin \text{dom}(\Gamma) \quad \Gamma \vdash v_{var} : string$

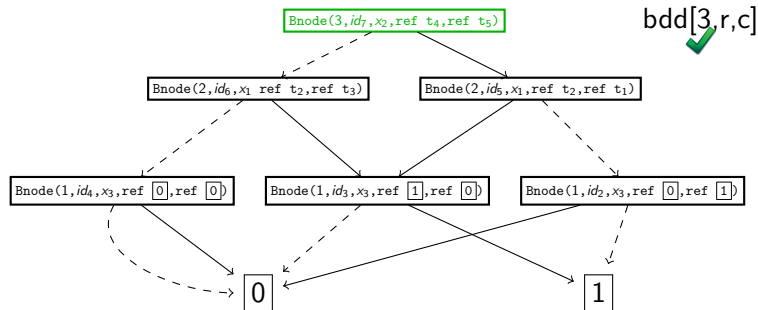
$\Gamma \vdash t_1 : \text{ref } \text{bdd}[l', r, c] \quad \text{bdd}[l', r, c] <:_B \{v : \text{bdd}[p, r, c] \mid l = p + 1\}$

$\Gamma \vdash t_2 : \text{ref } \text{bdd}[l'', r, c] \quad \text{bdd}[l'', r, c] <:_B \{v : \text{bdd}[p', r, c] \mid l = p' + 1\}$

T-BNODE

$\Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : \text{bdd}[l, r, c]$

# Type checking and type inference for a 3-level BDD



$$\begin{array}{c}
 \Gamma \vdash id : Id \quad id \notin \text{dom}(\Gamma) \quad \Gamma \vdash v_{var} : string \\
 \Gamma \vdash t_1 : \text{ref } bdd[l', r, c] \quad bdd[l', r, c] <:_B \{ \nu : bdd[p, r, c] \mid l = p + 1 \} \\
 \Gamma \vdash t_2 : \text{ref } bdd[l'', r, c] \quad bdd[l'', r, c] <:_B \{ \nu : bdd[p', r, c] \mid l = p' + 1 \} \\
 \hline
 \text{T-BNODE} \quad \Gamma, id : Id \vdash \text{Bnode}(t_0, id, v_{var}, t_1, t_2) : \text{bdd}[l, r, c]
 \end{array}$$

# BDDL in Action: Runtime Error Prevention in CUDD

```
DdNode* Cudd_bddCompose(  
... // bdd composition  
)  
  
int main () {  
  Cudd mgr(0,2);  
  BDD x = mgr.bddVar();  
  BDD y = mgr.bddVar();  
  BDD h = x * y;  
  BDD j = x + y;  
  BDD k = h.Compose(j,2); X  
}
```

⇒

```
letrec two : {ν : nat | ν = 2} =  
(succ (succ 0)) in  
letrec bddCompose : bdd[l, r, c]  
→ bdd[l', r, c] → {ν : nat | ν ≤ l - 1}  
→ bdd[l'', r, c] =  
  λ f .  
    λ g .  
      λ v .  
        <body>  
in  
letrec h = Bnode(two, ...) in  
letrec j = ... in  
letrec k =  
  bddCompose h j two ;
```

# BDDL in Action: Runtime Error Prevention in CUDD

```
DdNode* Cudd_bddCompose(  
... // bdd composition  
)  
  
int main () {  
  Cudd mgr(0,2);  
  BDD x = mgr.bddVar();  
  BDD y = mgr.bddVar();  
  BDD h = x * y;  
  BDD j = x + y;  
  BDD k = h.Compose(j,2);  
}
```



```
letrec two : { $\nu : \text{nat} \mid \nu = 2$ }  
  = (succ (succ 0)) in  
letrec bddCompose :  $\text{bdd}[l, r, c]$   
  →  $\text{bdd}[l', r, c]$   
  →  $\text{bdd}[l'', r, c]$  =  
  λ f .  
    λ g .  
      λ v .  
        <body>  
in  
letrec h :  $\text{bdd}[2, r, c]$   
letrec j :  $\text{bdd}[2, r, c]$   
letrec k =  
  bddCompose h j two
```

Runtime time error

$\{\nu : \text{nat} \mid \nu = 2\} \neq \{\nu : \text{nat} \mid \nu \leq 1\}$

Compile time error

# BDDL in Action: Runtime Error Prevention in MDDL

```
BddNode* Union_QQ(  
  BddNode *p, BddNode *q){  
  ASSERT((k = p->GetLevel())  
  == q->GetLevel());  
  ...  
  ka = answer->GetLevel();  
  ASSERT(ka == k);  
  return answer;  
}  
int main ()  
{  
  BddNode *f = new_bdd(1);  
  BddNode *g = new_bdd(2);  
  BddNode *res = Union_QQ(f,g);  
}
```

⇒

```
letrec union :  
  bbd[l, q, c]->  
  bbd[l, q, c] ->  
  bbd[l, q, c]) =  
λ p . λ r . <body>  
in letrec f : bbd[1, q, s] = ...  
in letrec g : bbd[2, q, s] = ...  
in union f g
```

# BDDL in Action: Runtime Error Prevention in MDDL

```
BddNode* Union_QQ(  
  BddNode *p, BddNode *q){  
  ASSERT((k = p->GetLevel()))  
  == q->GetLevel());  
  ...  
  ka = answer->GetLevel();  
  ASSERT(ka == k);  
  return answer;  
}  
int main ()  
{  
  BddNode *f = new_bdd(1);  
  BddNode *g = new_bdd(2);  
  BddNode *res = Union_QQ(f,g);  
}
```



```
letrec union :  
  bbd[l, q, c] -> X  
  bbd[l, q, c] -> X  
  bdd[l, q, c]) =  
  λ p . λ r . <body>  
in letrec f : bbd[1, q, s] X = ...  
in letrec g : bbd[2, q, s] X = ...  
in union f g  
bdd[1, q, s] ≠ bdd[2, q, s]
```

Runtime time error

Compile time error

# BDDL in Action: Runtime Error Prevention in JavaBDD

```
public class BasicTests
extends BDDTestCase {...
  public void testCrash() {
    reset();
    Assert.assertTrue(hasNext());
    BDDFactory bdd = nextFactory();
    BDD a = bdd.one();
    bdd.reorder(
      bdd.getReorderMethod());
  }
```

⇒

```
letrec n : bdd[0,r,c] = 1
in letrec reorder :
  { $\nu$  :  $bdd[l,r,c] \mid l \geq 1$ }
→ { $\nu$  :  $bdd[l,r,c] \mid l \geq 1$ } =
   $\lambda$  p :  $bdd[l,r,c]$ . <body>
in
  reorder n
```



# BDDL in Action: Runtime Error Prevention in JavaBDD

```
public class BasicTests
extends BDDTestCase {...
  public void testCrash() {
    reset();
    Assert.assertTrue(hasNext());
    BDDFactory bdd = nextFactory();
    BDD a = bdd.one();
    bdd.reorder(
      bdd.getReorderMethod());
  }
```

Runtime time error



```
letrec n : bdd[0,r,c] = 1
in letrec reorder :
  {ν : bdd[l,r,c] | l ≥ 1}
  → {ν : bdd[l,r,c] | l ≥ 1} =
    λ p : bdd[l,r,c]. <body>
in
  reorder n
  bdd[0,r,c] ≠ {bdd[l,r,c] | l ≥ 1}
```

Compile time error

# Our approach and future directions

## *Original library*

```
BddNode* UnionQQ  
( BddNode* p,  
  BddNode* q ){...}  
...
```

## *Annotated code*

```
@type union:  
bdd[l,q,c] -> bdd[l,q,c]  
-> {v: bdd[l',q,c] | l'=l}  
BddNode* UnionQQ  
( BddNode* p,  
  BddNode* q ){...}  
...
```


## *Type checker prototype in O'Caml*

```
letrec union:  
bdd[l,q,c] -> bdd[l,q,c]  
-> {v: bdd[l',q,c] | l'=l}  
...
```


*Manually constructed  
BDDL code*

# Some Related Work


- ▶ Static analysis

- ▶  [Lhoták, Ondřej, and Laurie Hendren](#)  
Jedd: a BDD-based relational extension of Java.  
[PLDI, 2004.](#)

- ▶ Runtime verification

- ▶  [Rolf Drechsler](#)  
Verifying integrity of decision diagrams.  
[Integration, the VLSI Journal, 2002.](#)

- ▶ Liquid types

- ▶  [Ming Kawaguchi, Patrick Rondon, and Ranjit Jhala](#)  
Type-based data structure verification.  
[PLDI, 2009.](#)

# Conclusion

