cs141 Workshop: Summations and Logarithms

Summations

Arithmetic Series:

$$\sum_{k=1}^{n} k = 1 + 2 + \cdots + n = \frac{1}{2}n(n+1) = \Theta\left(n^2\right)$$

Geometric Series:

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n = \frac{x^{(n+1)} - 1}{x - 1} = \Theta\left(x^n\right)$$

Harmonic Series:

$$\sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} = \ln n + O(1)$$

Linearity of summations:

$$\sum_{k=1}^{n}\left(ca_k + b_k\right) = c\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n} \Theta\left(f(k)\right) = \Theta\left(\sum_{k=1}^{n} f(k)\right)$$

Also know that you can:
- Bound unknown functions with summations and bounds of known functions.
- Add and subtract terms from summations that don't quite fit.

Examples:

1) Find $\displaystyle\sum_{i=1}^{n} 5i$

$$\sum_{i=1}^{n} 5i = 5\sum_{i=1}^{n} i = 5\frac{n(n+1)}{2} = \frac{5}{2}(n^2 + n) = \Theta\left(n^2\right)$$

2) Find $\displaystyle\sum_{i=3}^{n} i$.

3) Find $\displaystyle\sum_{i=0}^{n}\left(\frac{1}{2}\right)^i$ and $\displaystyle\sum_{i=1}^{n}\left(\frac{1}{2}\right)^i$

4) Find $\displaystyle\sum_{i=0}^{n} \Theta(n)$

5) Find $\displaystyle\sum_{i=1}^{n}\left(4(n^i + 5) + \frac{1}{i^2}\right)$

Logarithms

$$a = b^{\log_b a}$$

$$\log_c (ab) = \log_c a + \log_c b$$

$$\log_b a^n = n \log_b a$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log_b \left(\frac{1}{a} \right) = -\log_b a$$

$$\log_b a = \frac{1}{\log_a b}$$

$$a^{\log_b n} = n^{\log_b a}$$