cs141 Workshop: The Master Method

The Master Theorem:

Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence:

$$T(n) = aT(n/b) + f(n),$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then $T(n)$ can be bounded asymptotically as follows:

1. If $f(n) = O\left(n^{\log_b a - \varepsilon}\right)$ for some constant $\varepsilon > 0$, then $T(n) = \Theta\left(n^{\log_b a}\right)$.
2. If $f(n) = \Theta\left(n^{\log_b a} \log^k n\right)$ for some constant $k \geq 0$, then $T(n) = \Theta\left(n^{\log_b a} \log^{k+1} n\right)$.
3. If $f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right)$ for some constant $\varepsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta(f(n))$.

Use the master method (where applicable) to solve the following recurrence relations:

Assume that $T(n)$ is constant for $n \leq 2$.

1. $T(n) = 2T(n/2) + n^3$
2. $T(n) = T(9n/10) + n$
3. $T(n) = 16T(n/4) + n^2$
4. $T(n) = 7T(n/3) + n^2$
5. $T(n) = 7T(n/2) + n^2$
6. $T(n) = 2T(n/4) + \sqrt{n}$
7. $T(n) = T(n-1) + n$
8. $T(n) = T\left(\sqrt{n}\right) + 1$
9. $T(n) = 3T(n/2) + n \lg n$
10. $T(n) = 3T(n/3 + 5) + n/2$
11. $2T(n/2) + n/\lg n$
12. $T(n) = T(n-1) + 1/n$
13. $T(n) = T(n-1) + \lg n$
14. $T(n) = \sqrt{n}T\left(\sqrt{n}\right) + n$
15. $T(n) = 3T(n/4) + n$
16. $T(n) = 2T(n/2) + n$
17. $T(n) = 4T(n/2) + n^2$