1) Prove that all finite languages are regular by giving a general schema to construct an NFA for any finite language. (15 pts)

Construct an NFA for each string and add transitions for the initial and final states:

Since the language is finite, we are able to construct NFAs for each string.
2) Construct a DFA for the following language on $\Sigma = \{a, b\}$ (15 pts)
$L = \{w : n_a(w) \mod 3 > n_b(w) \mod 3\}$
3) Show that the language $L = \{a^n : n \geq 0, n \neq 4\}$ is regular. (15 pts)

4) Write a grammar which generates the following language: (Hint: find two separate grammars then concatenate them) (15 pts)

$$\{ww^n a^n b^{2n} : w \in \{a, b\}^*, n \geq 0\}$$

$$S_1 \rightarrow aS_1a | bS_1b | aa | bb \quad S_2 \rightarrow aS_2bb | \lambda$$

$$S \rightarrow S_1S_2$$
5) Consider the following NFA,
   a) For what reasons it's an NFA and not DFA? (3 pts)
   b) Convert it to an equivalent DFA. (10 pts)
   c) What is the algorithm called? (2 pts)

b) 1. Two 'a' transitions from q0
    2. λ-Transition
    3. Some Σ have missing transition

\[\begin{array}{c|ccc}
   & a & b & \lambda^* \\
\hline
q_0 & q_1, q_2 & q_0 & a, q_1, q_2, \lambda^* \\
q_1 & q_1, q_2 & q_1, q_2 & q_1, q_2 \\
q_2 & q_1, q_2 & q_1, q_3, q_2 & q_3, q_2 \\
q_3 & q_1, q_2 & q_1, q_2, q_3 & q_1, q_2, q_3 \\
\end{array}\]

\[\begin{array}{c|c}
   & a \lambda^* \\
\hline
q_0 & q_1, q_2 \\
q_1, q_2 & q_1, q_2 \\
q_1, q_2 & q_1, q_2, q_3 \\
q_1, q_2, q_3 & q_1, q_2, q_3 \\
\end{array}\]

\[\begin{array}{c|c}
   & b \lambda^* \\
\hline
q_0 & q_1, q_2 \\
q_1, q_2 & q_1, q_2 \\
q_1, q_2, q_3 & q_1, q_2, q_3 \\
q_1, q_2, q_3 & q_1, q_2, q_3 \\
\end{array}\]
6) Find a **left** linear grammar for the following automaton: (15 pts)

\[ S \rightarrow q_1 \mid q_0 \mid b \\
q_0 \rightarrow b q_2 \mid a q_1 \\
q_1 \rightarrow a q_0 \mid q_0 \mid b q_2 \mid q_2 \\
q_2 \rightarrow a q_2 \mid \lambda \]

\[ S \rightarrow q_1 \mid q_0 \]

\[ q_0 \rightarrow q_2 b \mid q_1 a \]

\[ q_1 \rightarrow q_0 a \mid q_0 \mid q_2 b \mid q_2 \]

\[ q_2 \rightarrow q_2 a \mid \lambda \]
7) Define $D(L) = \{ s_1, s_2 \mid s_1, s_2 \in L, \sigma \in \Sigma \}$. That is, $D(L)$ is the language of strings that can be obtained by deleting exactly one symbol from some string in $L$. Prove that if $L$ is regular then $D(L)$ is also regular. (15 pts)

If $L$ is regular there should exist an NFA for $L$. In order to construct an NFA for $D(L)$, make a copy from $L$ and add $\lambda$-transitions from $L$ to the copy of $L$ such that it skips only one of the transitions in $L$. Repeat this for all transitions in $L$. Then make the final states in $L$ to non-final states and keep the final states in the copy of $L$, unchanged. For example do the following:

- For loops, add an additional $\lambda$ transition from the state which has a loop in $L$ to the corresponding state in the copy of $L$ (example shown above) via the blue transition.
8) Regarding the Arden's Rule (which we discussed in the lecture),
   a) Describe the Arden's Rule. (5 pts)
   b) For the following automaton, find its corresponding regular expression by using the
      Arden's rule. (10 pts)

   ![Automaton Diagram]

   a) Regular Expression \( X = A \cdot X + b \Rightarrow X = A^*b \)  

   b) \[
   \begin{align*}
   q_0 &= aq_0 + bq_1 + bq_2 \\
   q_1 &= aq_2 + \lambda \Rightarrow q_1 = aqq_1 + a + \lambda \Rightarrow q_1 = (aa)^*(a+\lambda) \\
   q_2 &= aq_1 + \lambda \\
   \Rightarrow q_0 &= aq_0 + b(aa)^*(a+\lambda) + ba(aa)^*(a+\lambda) + b \\
   q_0 &= a^*(ba(aa)^*(a+\lambda) + ba(aa)^*(a+\lambda) + b)
   \end{align*}
   \]

9) It would be great to have your feedback so far on:
   a) The lecture (Me) (10 pts)
   b) The discussion session (Nick) (5 pts)

   (Thanks in advance)

   a) The lecture is so awesome :) JK
   b) astonishing TA :)