An Efficient Method for Transmission Line Simulation

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Abstract: In this paper, a method based SVD (Singular Value Decomposition) was used to cope with singular descript of transmission line model. In order to keep the passivity of the original interconnect model, PRIMA [1] and PRBT [2] were used to reduce the model for fast transmission line simulation.

Keyword: singular, MOR, SVD

1. Introduction

In this paper, we focus on the singularity of descriptor in state space equations. In general, a system is described by a normal state space equation. However, in interconnect simulation, by interconnect parameter extraction, modified nodal analysis (MNA) generates a state space with descriptor matrix E. Usually, E is singular, so it can not be changed to normal model by multiply its inversion. Our final objection is to reduce complexity of the original system, and use a low-order model to approximate the original system model with MOR methods.

MOR technique appear early in VLSI is moment matching, to approximate some frequency in transfer function, such as AWE, PRIMA [1], which have better numerical properties. Another technique stem from control theory, include passive preserving BT (balance truncation) [2], can preserve the stability passivity of system. However, the computation of solving algebraic Riccati equation for passive preserving balance truncation is horrible [2] [3].

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A general, reduced model generated by Krylov projection can not keep the passivity of original model, but PRIMA, can reduce the special model resulted by MNA without losing passivity. It is natural to find a way combining passivity preserve balance truncation with PRIMA to reduce order. The rest of this paper will introduce our scheme to realize it. The remainder is organized as follow: In Section 2, we simply review the PRIMA method. In Section 3, SVD method [4] will be represented to cope with singular descriptor. In Section 4, we introduce the relationship between passivity and positive real lemma. Section 5 gives some numerical results. Last Section make a conclusion.

2. Reduce model by PRIMA

When RCL interconnect parameters were extracted by tool, the interconnect model for simulation generated by MNA was below:

$$\begin{cases} E\dot{x} = Ax + Bu \\ y = Lx \end{cases} \tag{1}$$

where A and E represent the conductance and susceptance matrices respectively. And they have the construction as below:

$$A = -\begin{bmatrix} G & N \\ -N^T & 0 \end{bmatrix} E = \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} x = \begin{bmatrix} v \\ i \end{bmatrix} \tag{2}$$

where G, Q, H are the matrices containing the stamps for resistors, capacitors, and inductors, respectively. N consists of ones, minus ones, and zeros, which represent the current variables in KCL equations. And v, i denote the port voltages and current respectively. Provided that the original n ports is composed of passive linear elements only, C, L, and G are symmetric nonnegative definite matrices, then E is symmetric nonnegative definite matrix with this MNA formulation.

Assume A was moved from right to the left of first equation (1), we define:

$$H \equiv A^{-1}E, \quad R \equiv A^{-1}B \tag{3}$$

The remainder is solve the basis of Krylov space

$$K(H,R,q) = (R, HR, H^2R, \cdots, H^{q-1}R) \tag{4}$$

Using Arnoldi method, we can find a basis, denoted X, satisfied:

$$X = span(K(H,R,q)) \tag{5}$$

Applying the change of variable $x_n = Xx_q$ in (1), and multiply the first equation by X^T from (5) yields

$$(X^T EX)\dot{x}_q = (X^T AX)x_q + (X^T B)u$$

$$y = (LX)x_q \tag{6}$$

Then we get a reduced q-order model (6), it was proved that the reduced model preserve the passivity and its transform function $Y_q(s)$ matches $2q$ moments of the original $Y_n(s)$, detail see [1].

3. SVD for singular E

Usually, the model yielded by MNA has a singular matrix E. Sometimes, it was assumed that E is nonsingular to make an easy way to reduce model. However, we can not neglect the singularity appearing in the model of interconnect for simulation.

Although matrix E is changed to low dimension matrix E_q, however, the singularity still exist. That means some state variants are trivial in the reduced model. For decreasing computation, make singular value decomposition:

$$E_q = Q\Sigma Q^T \tag{7}$$

to get rid of them. Note that E_r is symmetric matrix, $Q^T Q = I_r$. And the elements of diagonal matrix Σ,

$$\sigma_1 \geq \cdots \geq \sigma_k > \sigma_{k+1} = \cdots = \sigma_q = 0 \tag{8}$$

Transform the r-order system by orthogonal matrix Q, a equal model can be get:

$$\begin{cases} \Sigma \dot{x}_q = Q^T A_q Q x_q + Q^T B_q u \\ y = L_q Q x_q \end{cases} \tag{9}$$

Notice, the state variants in (9) are not the ones in (6). And the new model is still passive system. Partition the system (9) as below:

$$\begin{cases} \begin{bmatrix} \Sigma_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \\ y = \begin{bmatrix} L_1 & L_2 \end{bmatrix} \begin{bmatrix} x_1^T & x_2^T \end{bmatrix}^T \end{cases} \tag{10}$$

Then we can remove the trivial state variants x_2 since $x_2 = -A_{22}^{-1}(A_{21}x_1 + B_2 u)$ and solve the descriptor problem by multiplying inverse of Σ_k both sides of first equation (10). The normal model is:

$$\begin{cases} \dot{x} = \hat{A}x + \hat{B}u \\ y = \hat{L}x + \hat{D}u \end{cases} \tag{11}$$

where

$$\hat{A} = A_{11} - A_{12}A_{22}^{-1}A_{21}$$
$$\hat{B} = B_1 - A_{12}A_{22}^{-1}B_2$$
$$\hat{L} = L_1 - L_2A_{22}^{-1}A_{21}$$
$$\hat{D} = -L_2A_{22}^{-1}B_2$$

4. Passivity Preserve BT

In this section, we firstly review the relationship between passivity of system and positive real lemma [3].

A system is passive if and only if there exists the solution a $P \geq 0$ satisfying the positive real lemma below:

$$\begin{aligned} A^T P + PA &= -Q^T Q \\ L^T &= PB + Q^T W \\ W^T W &= D + D^T \end{aligned} \quad (12)$$

We can determine the solution P by solving Riccati equation:

$$F^T P + PF + PXP + Y = 0 \quad (13)$$

where $F = A - BKL$, $X = BKB^T$ $Y = L^T KL$, $K = (D + D^T)^{-1}$. More detail see [3].

Let $W_o = P$ and substitute A^T , L^T and B by A, B and L^T respectively in (12), we get a similar equation with solution, denoted W_c . In fact W_c and W_o behaves as the controllability and observability gramians P, Q denoted as the solution of Lyapunov equations:

$$\begin{aligned} AP + PA^T + BB^T &= 0 \\ A^T Q + QA + C^T C &= 0 \end{aligned} \quad (14)$$

Similar to balance truncation for P and Q, find a balancing similarity transformation T, the state space model :

$$A \to T^{-1}AT, B \to T^{-1}B, L \to LT \quad (15)$$

and its gramians vary under the rules

$$\begin{aligned} &W_c \to T^{-1}W_c T^{-T}, W_o \to T^T W_o T \\ &W_c = W_o = \Sigma = diag(\sigma_1 \cdots \sigma_k) \end{aligned} \quad (16)$$

where $\sigma_1 \geq \cdots \geq \sigma_k \geq 0$. We may partition Σ into

$$\Sigma = \begin{bmatrix} \Sigma_1 & \\ & \Sigma_2 \end{bmatrix} \quad (17)$$

to delete those small elements. Then conformally partitioning the transformed matrices as

$$\hat{A} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix}, \hat{B} = \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix}, \hat{L} = \begin{bmatrix} \hat{L}_1 & \hat{L}_2 \end{bmatrix} \quad (18)$$

and truncating the model, retaining $\hat{A} = \hat{A}_{11}$, $\hat{B} = \hat{B}_1$, $\hat{L} = \hat{L}_1$ as the reduced system, therefore has the effect of deleting the smaller element of Σ . And the error in the transfer function of reduced model is bounded by

$$\| H(s) - H_r(s) \| \leq 2 \sum_{i=r+1}^{k} \sigma_i \quad (19)$$

5. Numerical Example

In this section, we test our algorithm with a practical transmission line model coming from benchmark in the website http://www.icm.tu-bs.de/NICONET/benchmodred.html. This is a 256-state model with a singular descriptor E and two input, two output system. For simple, we just cope with single input and single output with neglect other one. At first reduced order to 60-order by PRIMA, then using SVD to solve the singular E, finally reduced the second model got by SVD by passivity preserve BT method, and obtain the final 32-order model. Fig.1 shows they have a better approximation on frequency response. Fig. 2

show their Nyquist diagrams, we can find the reduced order keeps some characters of original system.

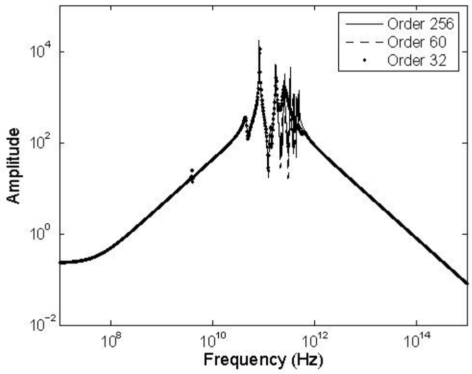

Fig. 1 Frequency Response

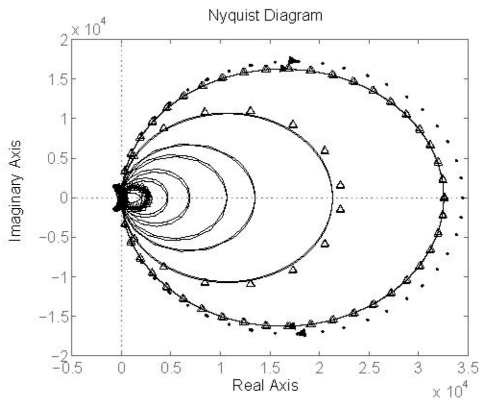

Fig. 2 Nyquist Frequency. Solid line is 256-order, triangle is 60-order, dot is 32-order model.

6. Conclusion

In this paper, we use some efficient methods to reduce the original model of transmission line, and make a better tradeoff on efficiency and accuracy. The future work is to study how to preserve some special characters in model reduction.

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